A comparison of Value–at–Risk methods for measurement of the financial risk[1]

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Abstract

One of the key concepts of risk measurements in financial sector and industrial sector is the probability-based risk measurement method known as Value-at-Risk or VaR. The results produced by a VaR model are simple for all levels of staff from all areas of an organisation to understand and appreciate. That is why VaR has been adopted so rapidly. We present some methods that use classical approach, and that uses copula approach for computing VaR, in this paper. The theory of copulas is known to provide a useful tool for modelling dependence in integrated risk management. Judicious choices of the method, with respect to the data used and computational aspect can be made to reduce the overall costs and computational time.

Key words: Value-at-Risk, copula function, correlation, Monte Carlo Analysis, historical simulation, delta-normal method

1 Introduction

Value-at-Risk (VaR), is a widely used measure of financial risk, which provides a way of quantifying and managing the risk of a portfolio. VaR was conceived in 1993 partly in response to various financial disasters. Work started on its development in 1988 after central banks wanted a methodology to set minimum capital requirements in banks to protect against credit risk in trading. Banks began adopting it around 1993-1995, as a key component of the management of market risk for many financial institutions[2]. It is used as an internal risk management tool, and has also been chosen by the Basel Committee on Banking Supervision as the international standard for external regulatory purpose[3]. Recent years, non-bank energy traders and end-users have begun to use VaR. Now the majority of major oil companies and traders are using the VaR method for risk measurement[4].

There are many approaches to measuring VaR. Any valuation model for computing VaR simply represent of a possible reality or a possible outcome, based on certain probability and confidence percentage parameters. VaR measures the worst expected loss over a given time horizon with a certain confidence – or probability – level. VaR allows management to see the probable risk their company is taking, or, in the case of companies hedging, it can also illustrate reduction in possible financial exposure. VaR can summarise all the market risks of the entire portfolio of a bank or a firm as one number for example in Slovak crowns (SKK).

The key use of VaR is for assessing market risk (exposure to losses in the market place through physical and derivative positions) although VaR is being used more frequently to assess credit risk (credit VaR modelling). However, VaR does not give a consistent method for measuring risk, as different VaR models will come up with different VaR results. It should also be noted that VaR only measures quantifiable risks; it cannot measure risks

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[2] Market risk is the risk associated with uncertainty about future earnings relating to changes in asset prices and market rates.

[3] The capital that a bank is required to hold against its market risk is based on VaR with a 10-day holding period at a 99% confidence level. Specifically, the ragulatory capital requirement for market risk is defined as $max(\mathrm{VaR}_{t-1}\ k \times \mathrm{Avg}\{\mathrm{VaR}_{t-i} |\ i=1, ..., 60\})$. Here k is multiplication factor, which is set to between 3 and 4 depending on previous backtest results, and VaR_t refers to a VaR estimate for day t based on a 10 day holding period [BAS96]

[4] James, T.: *Energy Price Risk: Trading and Price Risk Management.* Gordonsvile, VA, USA: Palgrave Macmillan, 2003, p.131.

such as liquidity risk, political risk, or regulatory risk. In times of great volatility, such as war, it may also not be reliable. For this reason, VaR models should always be used alongside stress testing[5].

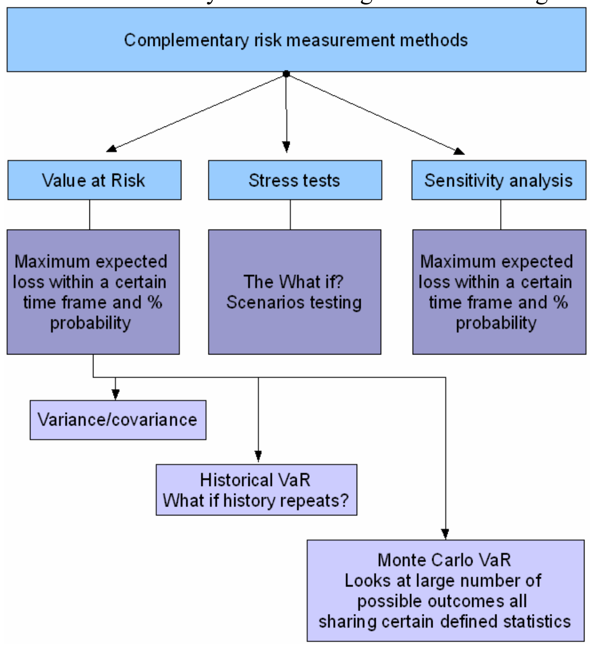

Figure 1: Risk measure methods[6]

Estimating the VaR of a portfolio involves determining a probability distribution for the change in the value of the portfolio over the time period (known as the holding period). The value of the portfolio of financial instruments, at time t depends on the k risk factors (market variables). These risk factors could be exchange rates, interest rates, stock prices, etc. Thus, the estimation VaR is done via estimation of the distribution of the underlying risk factors. The general techniques commonly used include analytic techniques:

1. parametric[7]
   - Delta-Normal method (local[8] valuation method),
   - Monte Carlo simulation (full[9] valuation method),
2. nonparametric[10]
   1. Historical Simulation.

The aim of this paper is briefly to describe and to compare these VaR methods on portfolios consisting of linear financial instruments - the government bonds – a British treasury strip – zero coupon bonds.

2 Delta-normal method

The delta-normal method is a parametric, analytic technique where the distributional assumption made is that the daily geometric returns of the market variables are multivariate normally distributed with mean return zero. Historical data is used to measure the major parameters: means, standard deviations, correlations. When the market value of the portfolio is a linear function of the underlying parameters, the distribution of the profits is normal as well. VaR is computed by multiplying the vector of first derivatives of the portfolio value with respect to the risk factor variables (the "deltas") by the specified covariance matrix, and then multiplying by a multiplier

[5] James, T.: *Energy Price Risk: Trading and Price Risk Management.* Gordonsvile, VA, USA: Palgrave Macmillan, 2003, p.133.
[6] James, T.: *Energy Price Risk: Trading and Price Risk Management.* Gordonsvile, VA, USA: Palgrave Macmillan, 2003, p.132
[7] Parametric techniques involve the selection of a distribution for the returns of the market variables, and the estimation of the statistical parameters of these returns [ENG03, p.7].
[8] In the local valuation method the distribution is estimated using a Taylor series approximation.
[9] The full valuation method generates a number of scenarios and estimates the distribution by revaluating a portfolio under these scenarios.
[10] Nonparametric techniques assume that the sample distribution is an adequate proxy for the population distribution [ENG03, p.7].

that depends on the normal distribution quantile point for the confidence level at which VaR is being computed (1.65 σ bellow the mean give 5 % level, 2.33 σ will give the 1 % level). This method was introduced by the J.P.Morgan's RiskMetrics™ system (http://www.jpmorgan.com or http://www.riskmetrix.reuters.com). The detailed description of this method we can found in [JOR00, p.206-221] or [ENG03, p.18-20]. Figure 2 details the step, involves in this approach.

The advantages of this method include its speed and simplicity, and the fact that distribution of returns need not be assumed to be stationary through time, since volatility updating is incorporated into the parameter estimation.

The delta-normal method can be subject to a number criticism. A first problem is the existence of *fat tails* in the distribution of returns on most financial assets. The distribution of daily returns of any risk factor would in reality typically show significant amount of positive kurtosis. This leads to fatter tails and extreme outcomes occurring much more frequently than would be predicted by the normal distribution assumption, which would lead to an underestimation of VaR (since VaR is concerned with the tails of the distribution). Another problem is that the method inadequately measures the risk of *nonlinear instruments*, such as options or mortgages.

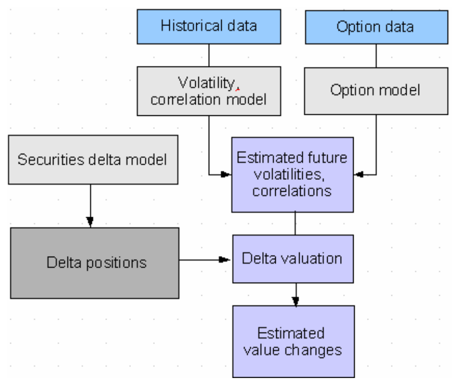

Figure 2: Delta-Normal method[11]

3 Historical simulation

Historical simulation method provides a straightforward implementation of full valuation (Figure 3). The simulated market states are produced by adding to the base case the period-to-period changes in market variables in the specified historical time series.

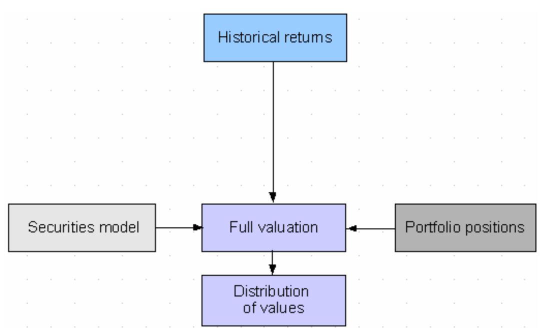

Figure 3:Historical simulation method[12]

The key assumption in historical simulation is that the set of possible future scenarios is fully represented by what happened over a specific historical window. This methodology involves collecting the set of risk factor changes over a historical window: for example, daily changes over the last five years. The set of scenarios thus obtained is assumed to be a good representation of all possibilities that could happen between today and tomorrow. The instruments in the portfolio are then repeatedly re-valued against each of the scenarios. This produces a distribution of portfolio values, or equivalently, a distribution of changes in portfolio value from

[11] Jorion, P.: *Value at Risk: The Benchmark for Controlling Market Risk.* Blacklick, OH, USA: Mc Graw-Hill Professional Book Group, 2000, p.220.
[12] Jorion, P.: *Value at Risk: The Benchmark for Controlling Market Risk.* Blacklick, OH, USA: Mc Graw-Hill Professional Book Group, 2000, p.222.

today's value. Usually, some of these changes will involve profits and some will involve losses. Ordering the changes in portfolio value from worst to best, the 99% VaR, for example, is computed as the loss such that 1% of the profits or losses are below it, and 99% are above it.

The main advantage of historical simulation is that it makes no assumptions about risk factor changes being from a particular distribution. Therefore, this methodology is consistent with the risk factor changes being from any distribution. Another important advantage is that historical simulation does not involve the estimation of any statistical parameters, such as variances or covariances, and is consequently exempt from inevitable estimation errors. It is also a methodology that is easy to explain and defend to a non-technical and important audience, such as a corporate board of directors.

However, as is usually the case, this methodology also has some disadvantages. The most obvious disadvantage is that historical simulation, in its purest form, can be very difficult to accomplish because it requires data on all risk factors to be available over a reasonably long historical period in order to give a good representation of what might happen in the future. Another disadvantage is that historical simulation does not involve any distributional assumptions, the scenarios that are used in computing VaR are limited to those that occurred in the historical sample. The next section describes how Monte Carlo simulation can be used to address this limitation of historical simulation.

4 Monte Carlo simulation method

Monte Carlo simulation techniques are by far the most flexible and powerful, since they are able to take into account all non-linearities of the portfolio value with respect to its underlying risk factor, and to incorporate all desirable distributional properties, such as fat tails and time varying volatilities. Also, Monte Carlo simulations can be extended to apply over longer holding periods, making it possible to use these techniques for measuring credit risk. However, these techniques are also by far the most expensive computationally.

The key difference between historical simulation and simulation Monte Carlo is that the historical simulation model carries out the simulation using the real observed changes in the market place over the last *X* periods (using historical market price data) to generate *Y* hypothetical portfolio profits or losses, whereas in the Monte Carlo simulation a random number generator is used to produce tens of thousands of hypothetical changes in the market. These are then used to construct thousands of hypothetical profits and losses on the current portfolio, and the subsequent distribution of possible portfolio profit or loss. Finally, the VaR is determined from this distribution according to the parameters set (e.g. 95 % confidence level). The method is summarized in Figure 4.

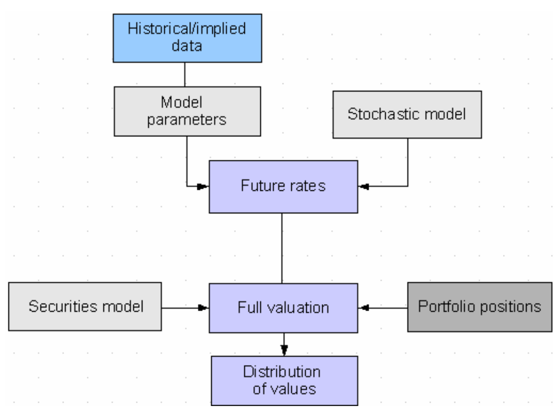

Figure 4:Monte Carlo method[13]

The ***RiskMetrics Monte Carlo*** methodology consists of three major steps[14]:

- Scenario generation, using the volatility and correlation estimates for the underlying assets in our portfolio, we produce a large number of future price scenarios in accordance with the lognormal models.
- For each scenario, we compute a portfolio value.
- We report the results of the simulation, either as a portfolio distribution or as a particular risk measure.

[13] Jorion, P.: *Value at Risk: The Benchmark for Controlling Market Risk.* Blacklick, OH, USA: Mc Graw-Hill Professional Book Group, 2000, p.225.
[14] Graeme West: *Risk Measurement for Financial Institutions.* www.smealsearch.psu.edu/29776.html

Other Monte Carlo methods may vary the first step by creating returns by (possibly quite involved) modeled distributions, using pseudo random numbers or quasi random numbers to draw a sample from the distribution. The next two steps are as above. The calculation of VaR then proceeds as for the historical simulation method.

The advances in other Monte Carlo methods over RiskMetrics Monte Carlo are in the creation of the distributions. However, to create experiments using a Monte Carlo method is fraught with dangers. Each market variable has to be modeled according to an estimated distribution and the relationships between distributions (such as correlation or less obvious non-linear relationships, for which copulas are becoming prominent). Using the Monte Carlo approach means one is committed to the use of such distributions and the estimations one makes. These distributions can become inappropriate; possibly in an insidious manner. To build and keep current a Monte Carlo risk management system requires continual reestimation, a good reserve of analytic and statistical skills, and non-automatic decisions.

One of these advanced methods use ***dynamic risk factor modeling***[15]. Dynamic risk factor modeling enables risk management practitioners to efficiently fit many risk factor models simultaneously. Risk factors models can be fitted using different distributional specifications, including non-normal distributions. This multivariate simulation process captures and maintains the dependence structure of the risk factors modelled separately. To accomplish this, the simulation engine uses a framework based on the statistical concept of a copula. A copula is a function that combines marginal distributions of the variables (risk factors) into a specific multivariate distribution in which all of its one dimensional marginals are the cumulative distribution functions (CDFs) of the risk factors. This dynamic approach to risk factor modeling is appropriate for market and for credit risk applications.

5 Results

The VaR methods, described above, will be tested for hypothetical portfolios of government bonds. We are holding 10 of these government bonds that mature in one month. The bonds are foreign instruments in our portfolio. The risk factors are the foreign exchange rate (British Pounds Sterling (GBP) to Slovak crown (SKK)) and the one-month interest rate LIBOR.

Although the Delta-Normal method and Monte Carlo simulation are parametric, and Historical simulation is nonparametric, direct comparison is possible since the distributional parameters will be estimated using the same historical data as will be used to generate scenarios for the Historical simulation method. We use SAS® Risk Dimensions® software[16] for this purpose.

VaR was estimated for this portfolio at the 90%, 95% and 99% confidence levels. The distributions of the profit/loss of our portfolio we see on **Figure 5** and Figure 6. The results VaR of as percent of Base Value of the portfolio are summarized in the Table 1.

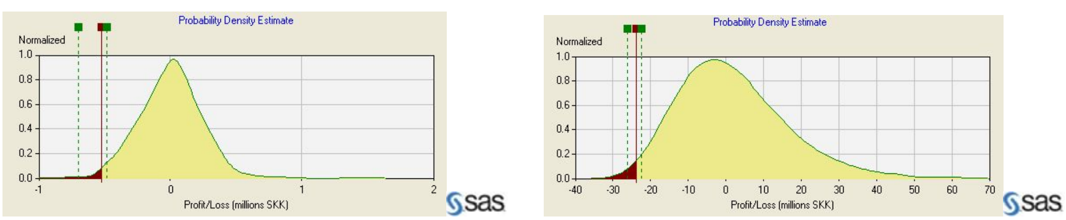

Figure 5: 1 day 99% VaR obtained by Historical simulation and by Monte Carlo simulation (RiskMetrics approach)

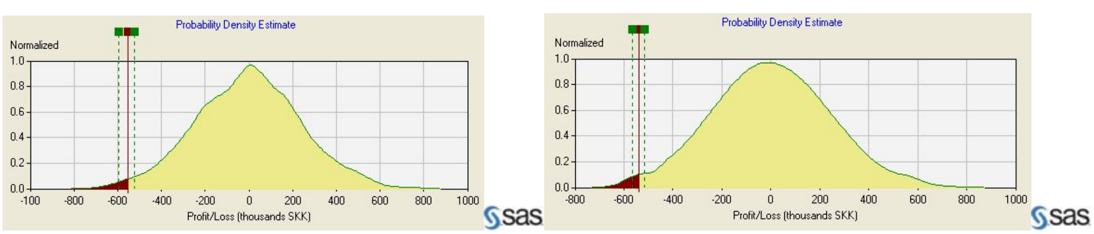

Figure 6: 1 day 99% VaR obtained by Monte Carlo simulation for pseudo random numbers
and by Monte Carlo simulation for Faure's quasi random number (both - dynamics approach)

[15] SAS® RISK Dimensions®: *Dynamic Risk factor Modeling Methodology*. White Paper, available on http:www.riskadvisory.com/pdfs/sasriskdimensionsriskfactor. pdf

[16] SAS® Risk Dimensions® software was financed to aid grant that is sponsored by Tatra banka Slovakia and to aid SAS Institute Slovakia.

Mark to Market Value (SKK)	VaR	Delta-normal method	Historical Simulation	Monte Carlo RiskMetrics	Monte Carlo (dynamic modelling) pseudo random number	Monte Carlo (dynamic modelling) Faure's quasi random number
53993907.96	1%	59.21	0.97	48.02	1.08	1.08
	5%	41.86	0.73	36.01	0.74	0.77
	10%	32.62	0.56	28.73	0.59	0.57

Table 1: At-Risk Value as percent of Base Value

The main advantages of VaR as a risk measure are well known. VaR provides a consistent measure of risk across all types of positions and across all kinds of markets and risk factors. For example, a VaR number for a fixed income position can be meaningfully compared to a VaR number for an equity position if they have been computed by using the same assumptions. Although this might seem obvious, until VaR gained acceptance, there was not such a measure that was in wide use. For example, measures such as duration and convexity were used for fixed income, and standard deviations were used for equities. Another advantage is that VaR can take into account interrelationships between different risk factors. Generally, a risk factor is anything that impacts portfolio value and would take a stochastic that is non-deterministic, path in the future. This capability can range from making use of simple correlations to making use of more subtle interrelationships, depending on the methodology that is used. In addition to risk reporting, VaR can be used in a variety of ways in an enterprise, such as setting limits or risk targets at various levels of the enterprise, for capital allocation at various levels including firm-wide capital, for comparing risks of deals before they are finalized, and for risk-adjusted performance measurement at various levels of the enterprise. Thus, it is important to recognize that VaR can be used effectively as a strategic tool and is not merely a regulatory requirement.

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